｜賴志煌教授：量子物理講義｜

【Chapter 4 Atomic structure】

Thomson model of the atom

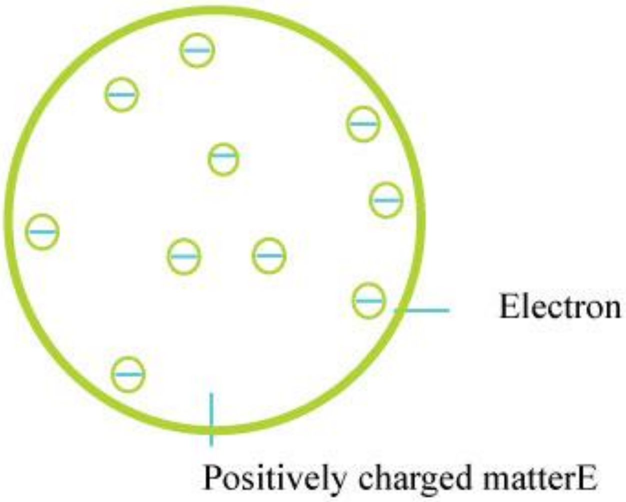


Figure 4.1 The Thomson model of the atom.

The Rutherford scattering experiment showed it to be incorrect.

Geiger and Marsden’s experiment:

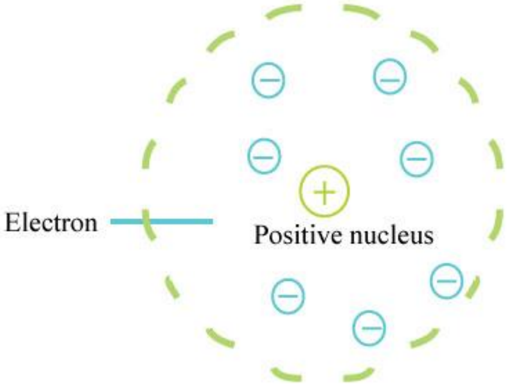


Figure 4.2 The Rutherford scattering experiment.

According to Thomson model, it was expected that the alpha particle (helium atoms lost two e', leaving charge of +2e) would go right through the foil with hardly any deflection. (Since the electron charge inside an atom is uniformly spread out its volume, only weak electric forces can exert on the alpha particle)

Although most of the alpha particles indeed were not deviated by much, a few were scattered through very large angles. Some were even scattered backwards.

Rutherford model

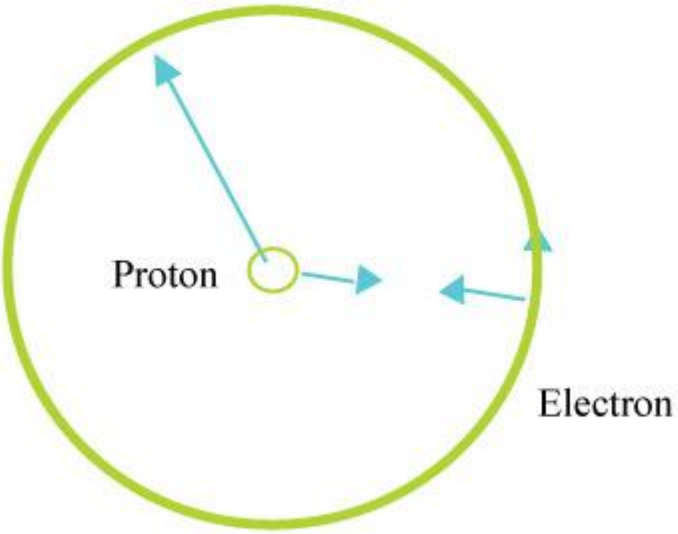


Figure 4.3 The Rutherford model of the atom.

With an atom being largely empty space, it is easy to see why most alpha particles go right through a thin foil. However, when an alpha particle happens to come near a nucleus, the intense E field scatters it through a large angle. The e' is do light that they do not appreciable affect the alpha particles.

The deflection of an alpha particle depends on magnitude of the nuclear charge.

All the atoms of any one element turned out to have the same unique nuclear charge, and this charge increased regularly in the periodic table.

The nuclear charges turned out to be multiples of +e, the number Z in the nuclei of an element is called **atomic number** of the element.

In fact, the proton with charge +e provide the charge on a nucleus, so the atomic number of an element is the same as the number of protons in the nuclei of its atoms.

Rutherford scattering formula

$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_o)^2 r^2 KE^2 \sin^4(\theta/2)}$$

N(θ) number of alpha particles per unit area that reach the screen at a scattering angle ofθ

Ni=total number of alpha particles that reach the screen

n=number of atoms per unit volume on the foil

Z=atomic number of the foil atoms

R=distance of screen from the foil

KE=kinetic energy of the alpha particles

t=foil thickness

Because N(θ) is inversely proportional to $\sin^4(\theta/2)$: only 0.14 percent of the incident alpha particles are scattered by more than 1^o.

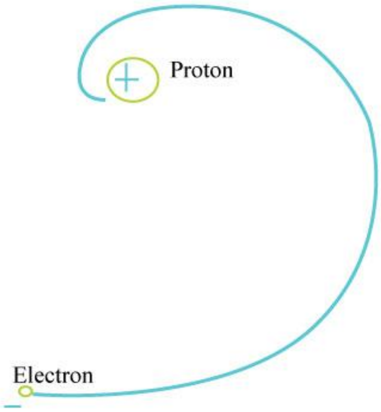


Figure 4.4 Rutherford scattering N(θ) is the number of alpha particles per unit area that reach the screen at a scattering angle ofθ; N(180 °) is this number for backward scattering. The experimental findings follow this curve, which is based on the nuclear model of the atom.

Nuclear Dimensions

Rutherford assumed that the size of a target nucleus is small compared with the minimum distance R to witch incident alpha particles approach the nucleus before being deflected away. ⟶ A way to find an upper limit to nuclear dimensions.

Smallest R=when alpha particles approach a nucleus head on, which will be followed by a 180^o scattering.

$KE = PE = \frac{1}{4\pi\varepsilon_o}\frac{2Ze^2}{R}$ ($\because$ αparticle 2+e nucleus Z+e)

$R = 2Ze^2/(4\pi\varepsilon_o KE) = 3.8x10-16\ Z(m)$ (for KE ofαparticle=7.7Mev)

For gold . Z=79 ⟶ $R(Au)=3x10^{-14}m$ ($<<10^{-4}$the radius of atom)

Electron orbits

e' cannot be stationary in this model, because there is nothing that can keep them in place against the electric force pulling them to the nucleus ⟶ the e' needs to be in motion

Assume a circular electron orbit ⟶

$F_c = mv^2/r$ (centripetal force)

$F_e = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}$ (electric force)

$$F_c = F_e \longrightarrow mv^2/r = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2} \longrightarrow v = \frac{e}{\sqrt{4\pi\varepsilon_o mr}}$$

Total energy $E = KE + PE = (1/2)mv^2 + \left(-\frac{e^2}{4\pi\varepsilon_o r}\right)$

$$= \frac{e^2}{8\pi\varepsilon_o r} - \frac{e^2}{4\pi\varepsilon_o r}$$

Total energy of H atom $E = -\frac{e^2}{8\pi\varepsilon_o r}$ (negative energy ⟶ e' is bound to nucleus)

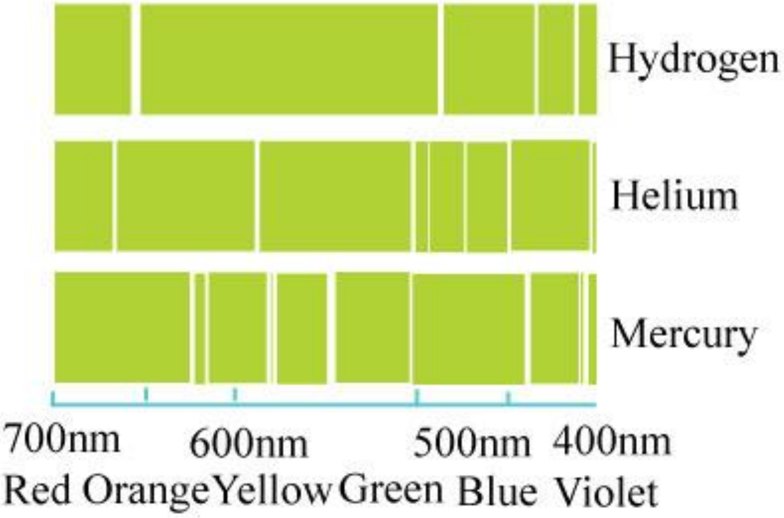


Figure 4.5 Force balance in the hydrogen atom.

The failure of classical physics

According to EM theory, the accelerated electric charges radiate energy in the form of em waves. An electron pursuing a curved path is accelerated and therefore should continuously lose energy, spiraling into the nucleus in a fraction of a second.

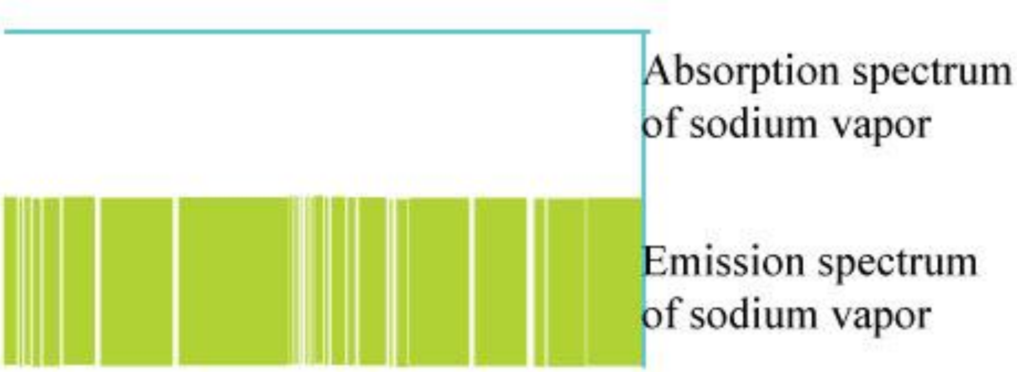


Figure 4.6 An atomic electron should, classically, spiral rapidly into the nucleus as it radiates energy due to its acceleration.

Classical physics fails to provide a meaningful analysis of atomic structure because it approaches nature in terms of “pure” particles and “pure” waves.

The usefulness of classical physics decreases as the scale of the phenomena under study decrease. ⟶ We must use the particle behavior of waves and the wave behavior of particles to understand the atom. ⟶ Bohr model

Atomic spectra

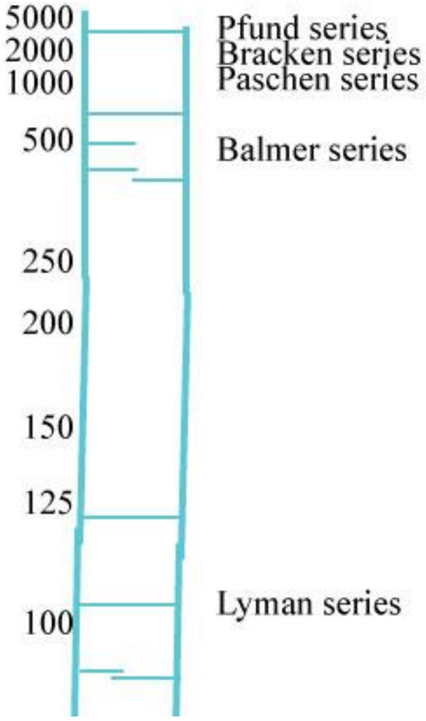


Figure 4.7 An idealized spectrometer.

Figure4.8 Some of the principal lines in the emission spectra of hydrogen, helium, and mercury.

Emission line spectra

Every element displays a unique line spectrum when a sample of it in the vapor phase is excited. Spectroscopy is useful tool for analyzing the composition of an unknown substance.

Absorption line

When white light is passed through a gas, the gas is found to absorb light of certain of the wavelength present in its emission spectrum.

Figure 4.9 The dark lines in the absorption spectrum of an element correspond to bright lines in its emission spectrum.

The number, strength, and wavelengths of the lines depend on temperature, pressure, the presence of electric and magnetic field, and the motion of the source.

Spectral series

Balmer series $1/\lambda=R(1/2^2 - 1/n^2)$ n=3,4,5……..

Rydberg constant $R=1.097\text{x}10^7\text{m}^{-1}=0.01097\text{nm}^{-1}$

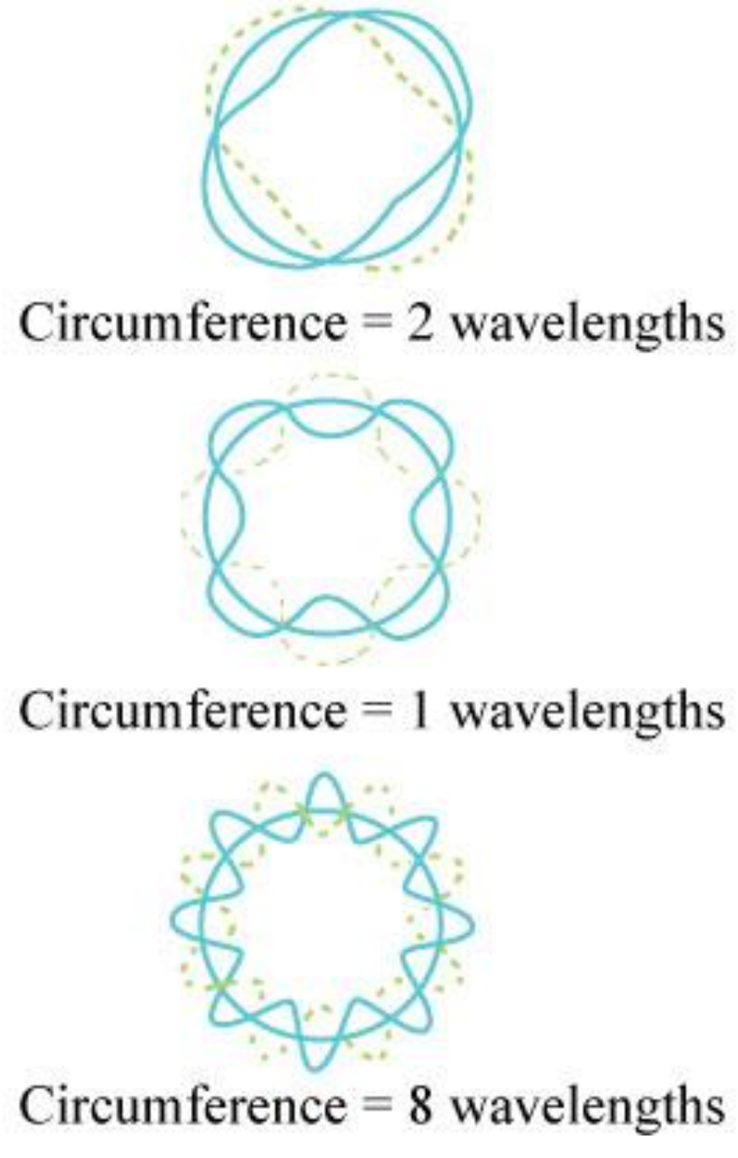


Figure 4.10 The Balmer series of hydrogen. The H_α lines is red, the H_β lines is blue, the H_γ and H_δ lines Are violet, and the other lines are in the near ultraviolet.

Lyman series	$1/\lambda=R(1/1^2-1/n^2)$	n=2,3,4…….
Pachen series	$1/\lambda=R(1/3^2-1/n^2)$	n=4,5,6……..
Brackett series	$1/\lambda=R(1/4^2-1/n^2)$	n=5,6,7……..
Pfund series	$1/\lambda=R(1/5^2-1/n^2)$	n=6,7,8………

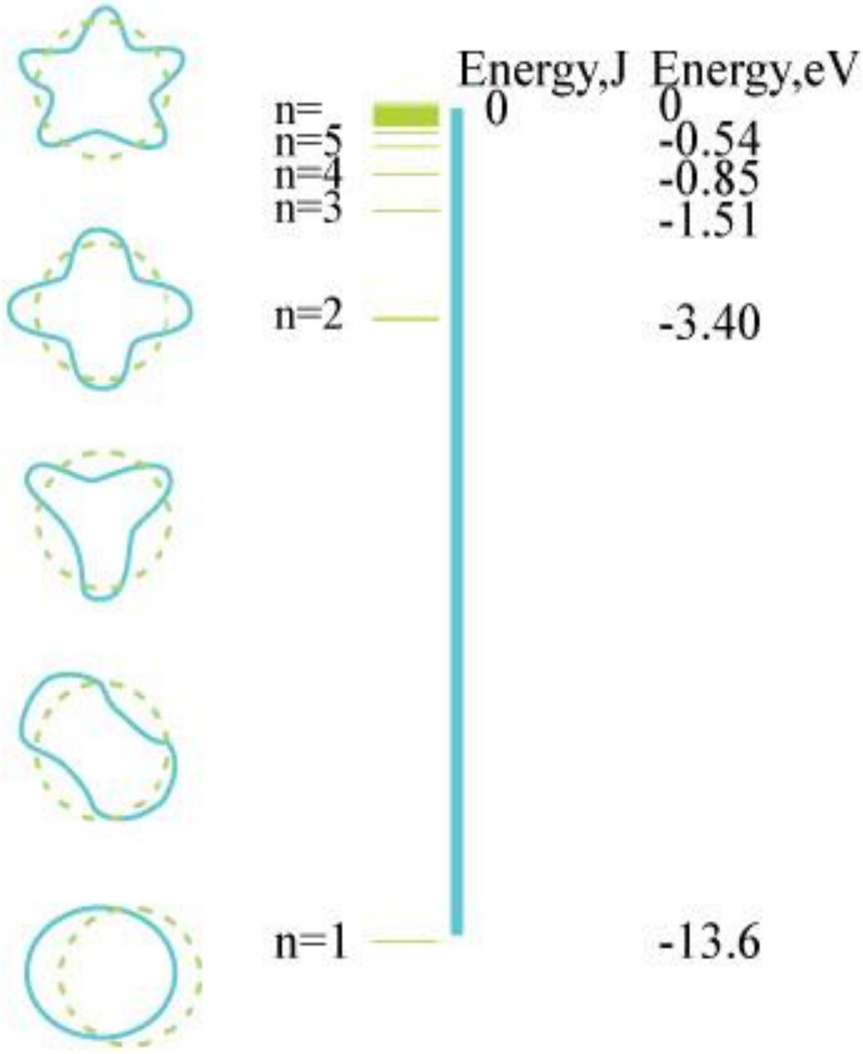


Figure 4.11 The spectral series of hydrogen. The wavelengths in each series are related by simple formulas.

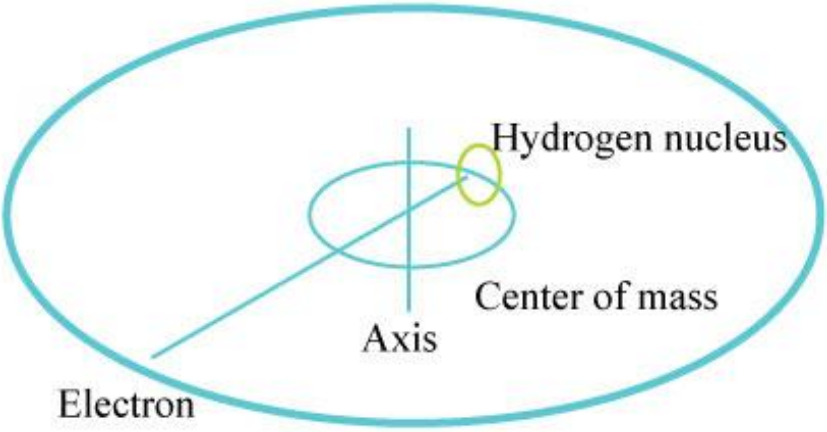


Figure 4.12 The orbit of the elec-tron in a hydrogen atom corre-sponds to a complete electron de Broglie wave joined on itself.

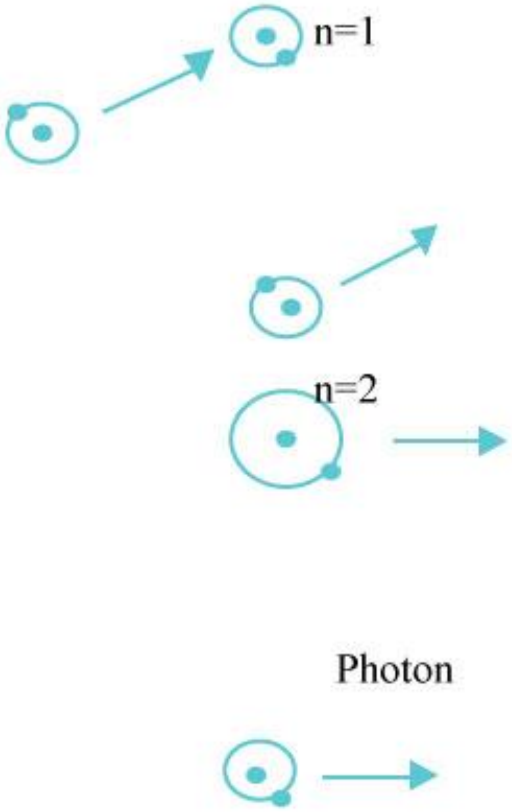


Figure 4.13 Some models of vi-bration of a wire loop. In each case a whole number of wave-lengths fit into number of wave-lengths fit into the circumference of the loop.

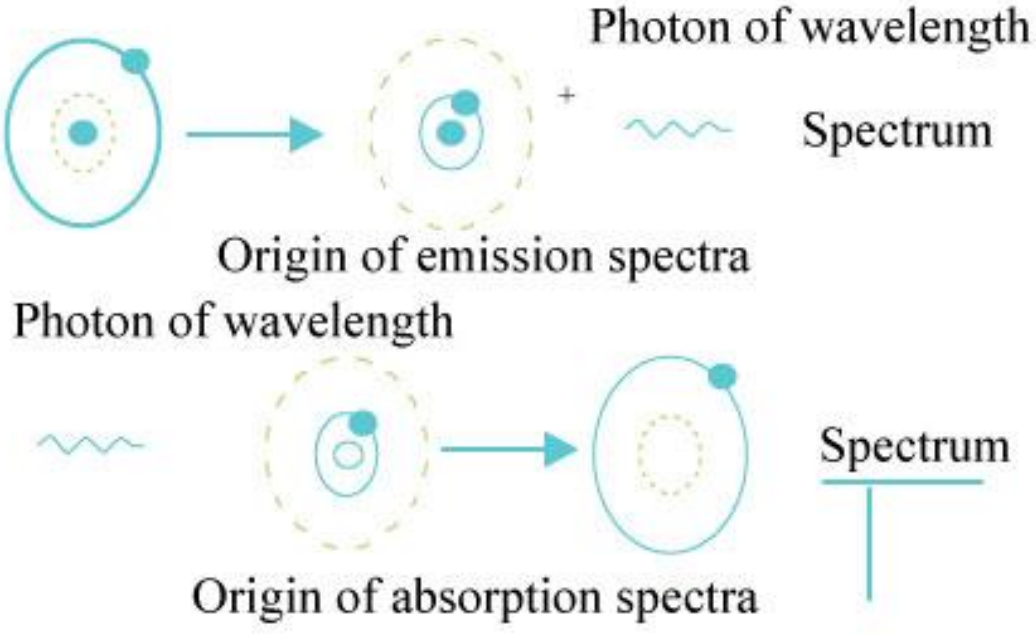


Figure 4.14 A fractional number of wavelengths cannot persist because destructive interference

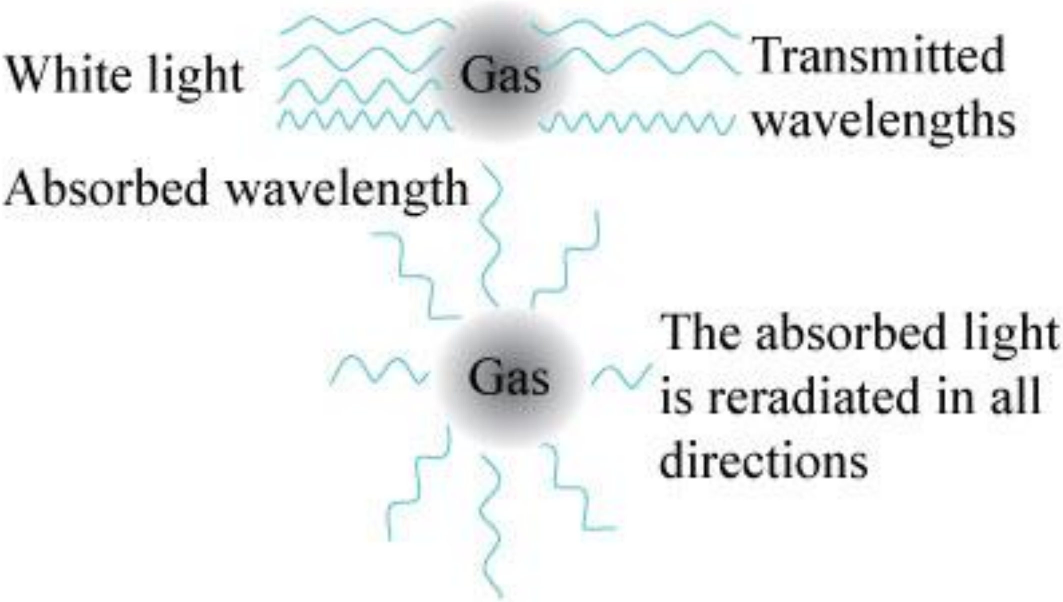


Figure 4.15 Energy levels of the hydrogen atom.

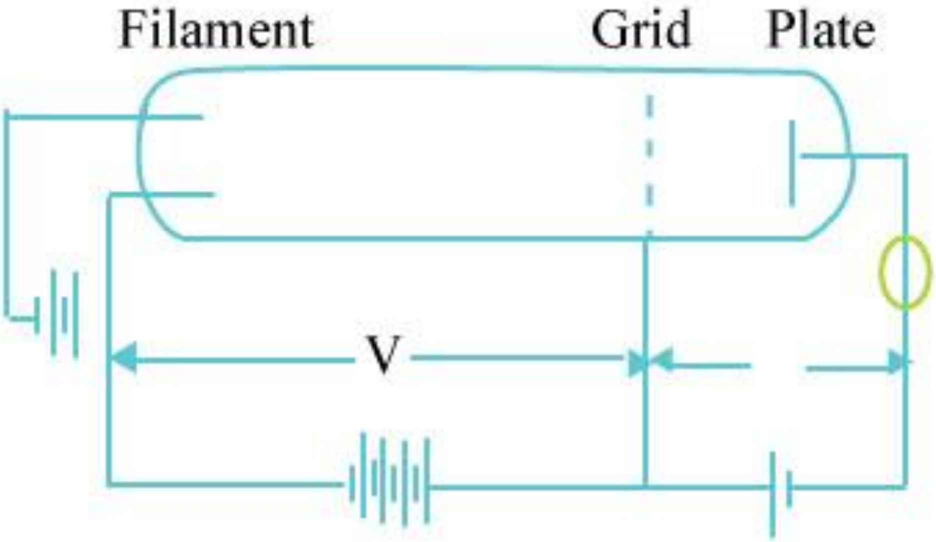


Figure 4.16

【4.4 Bohr atom】

de Broglie wavelength of e' $\lambda = h/mv$

$$v = \frac{e}{\sqrt{4\pi\varepsilon_o rm}} \longrightarrow \lambda = \frac{h}{e}\sqrt{\frac{4\pi\varepsilon_o r}{m}}$$

let $r = 5.3\text{x}10^{-11}$m $\longrightarrow$ $\lambda = 33\text{x}10^{-11}$m

$\lambda$is exactly the same as $2\pi r$

*An electron can circle a nucleus only if its orbit contains an integral number of de Broglie λ

$\longrightarrow$ $n\lambda = 2\pi r_n$ n=1,2,3……..quantum number

$$\longrightarrow \quad n\frac{h}{e}\sqrt{\frac{4\pi\varepsilon_o}{m}} = 2\pi r_n$$ (see Fig 4.12,4.13,4.14)

Orbital radii in Bohr atom $\longrightarrow$ $r_n = \frac{n^2 h^2 \varepsilon_o}{\pi m e^2}$ n=1,2,3…..

When n=1 , the radius is called Bohr radius a_o

$a_o = 5.292\text{x}10^{-11}$m , $r_n = n^2 a_o$

【4.5 energy level and spectron】

$\because$ $E_n = \frac{e^2}{8\pi\varepsilon_o r_n}$ $\longrightarrow$ $E_n = \frac{-me^4}{8\varepsilon_o{}^2 h^2}\left(\frac{1}{n^2}\right) = \frac{E_1}{n^2}$

E_1=-13.6ev(see fig 4.15)

These levels are all "-", which signifies that the e' does not have enough energy to escape from the nucleus.

E_1 : ground state

E_2,E_3,E_4……: excited state

- The work needed to remove an electron from an atom in its ground state is called its ionization energy.The ionization energy = - E_1

When an electron in an excited state drops to a lower state, the lost energy is emitted as a lingle of light.

$\longrightarrow$ $E_i - E_f = h\nu$

$E_1 (1/n_i^2 - 1/n_f^2) = -E_1(1/n_f^2 - 1/n_i^2) = h\nu$

$\nu = -E_1/h(1/n_f^2 - 1/n_i^2)$

$\rightarrow$ $1/\lambda = (-E_1/ch)(1/n_f^2 - 1/n_i^2)$ (see fig 4.16)

the constant $\frac{E_1}{ch} = \frac{me^4}{8\varepsilon_o{}^3 ch^3} = 1.097\times10^7 m^{-1}$ =Rydberg constant

【4.6 Correspondence Principle】

→ the greater the quantum number, the closer quantum physics approaches classical physics.

According to EM theory, an e' moving in a circular orbit radiates em waves whosevare equal to itsvof revolution and to harmonics(integral multiples) of that frequency.

In H atom, $v = \dfrac{e}{\sqrt{4\pi\varepsilon_o m r}}$

Frequency of revolution $f = v/2\pi r = \dfrac{e}{2\pi\sqrt{4\pi\varepsilon_0 m r^3}}$

$$\because \quad r_n = \frac{n^2h^2\varepsilon_o}{\pi m e^2} \longrightarrow \quad f = \frac{me^4}{8\varepsilon_0{}^2h^3}\left(\frac{2}{n^3}\right) = \frac{-E_1}{h}\left(\frac{2}{n^3}\right)$$

- From the Bohr theory

$$\nu = \frac{-E_1}{h}\left(\frac{1}{n_f{}^2} - \frac{1}{n_i{}^2}\right)$$

let $n_i = n$, $n_f = n\text{-}P$ (P=1,2,3……)

$$\longrightarrow \quad \nu = \frac{-E_1}{h}\left(\frac{1}{(n-p)^2} - \frac{1}{n^2}\right) = \frac{-E_1}{h}\left[\frac{2np - p^2}{n^2(n-p)^2}\right]$$

when n_i & n_f are both very large → n>>P

and $2np\text{-}p^2 \cong 2nP$ $(n\text{-}P)^2 \cong n^2$

$$\longrightarrow \quad \nu = \frac{-E_1}{h}\left(\frac{2p}{n^3}\right) \longrightarrow \text{same as classical em theory}$$

【4.7 Nuclear Motion】

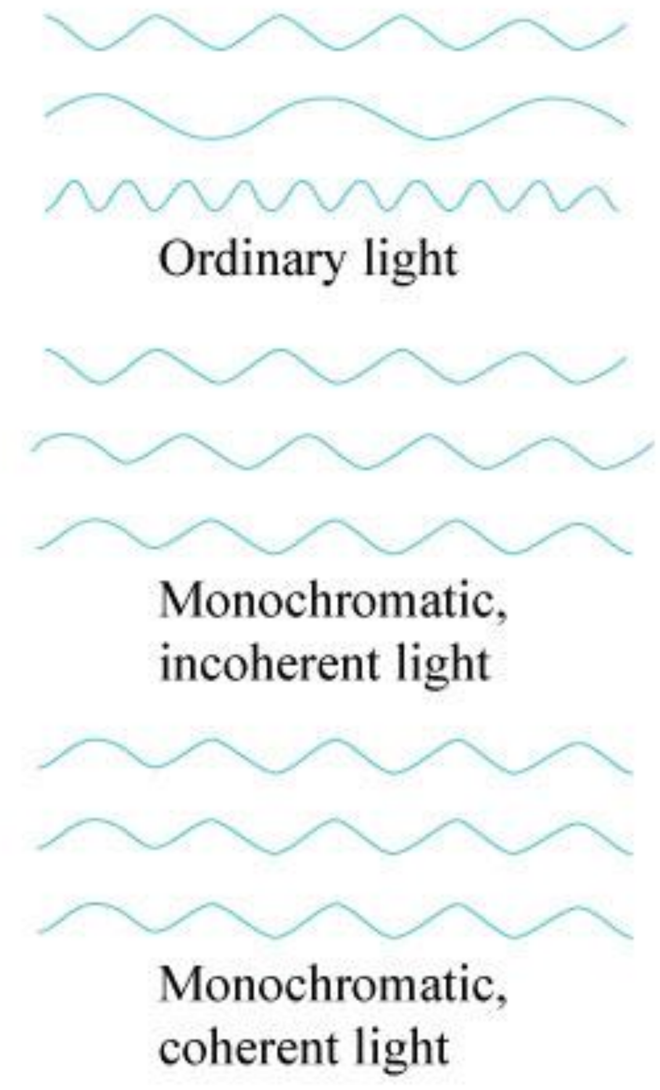


Figure 4.17 Both the electron and nucleus of a hydrogen atom revolve around a common center of mass.

The nuclear mass effects the λ of spectral lines.

Both nucleus & e' revolve around their common center of mass, which is very close to the nucleus because nuclear mass is much greater than that of the e' .

→ a single particle of mass m' revolves around the position of the heavier particle.

→ Reduced Mass m'=mM/m+M

$$E_n{}' = \frac{-m'e^4}{8\varepsilon_o{}^2h^2}\frac{1}{n^2} = \left(\frac{m'}{m}\right)\left(\frac{E_1}{n^2}\right)$$

In H m'/m=0.99945

Increase of 0.055% (∵E is ”-“)

Reduced mass deuterium →

$M_D = 2M_H$ (neutron + proton)

∵ mass increases → spectral lines of D shifted to shorter wavelength.

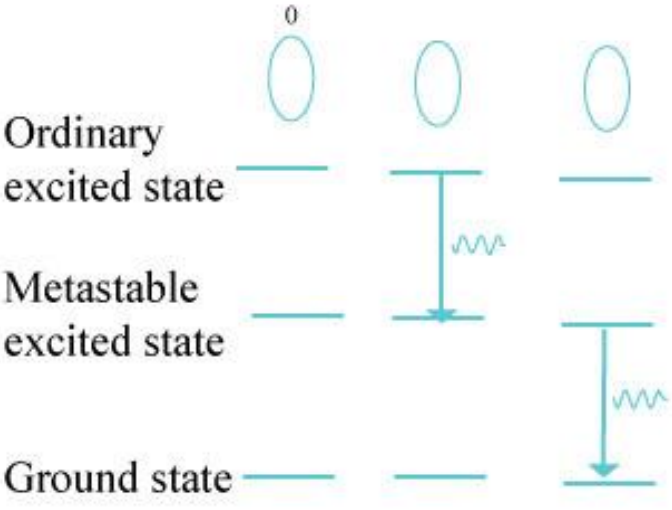


Figure 4.18 Excitation by collision. Some of the available energy is absorbed by one of the atoms, which goes into an excited energy state, The atom then emits a photon in returning to its ground (normal) state.

【4.8 Atomic excitation】

Two ways for an atom to be excited

(1) collision with another particle (KE is absorted)

excited atom → ground state (10^{-8}sec)
↓
hν

(2)atoms absorbs a photon of light whose energy is just the right amount to raise the atom → high level

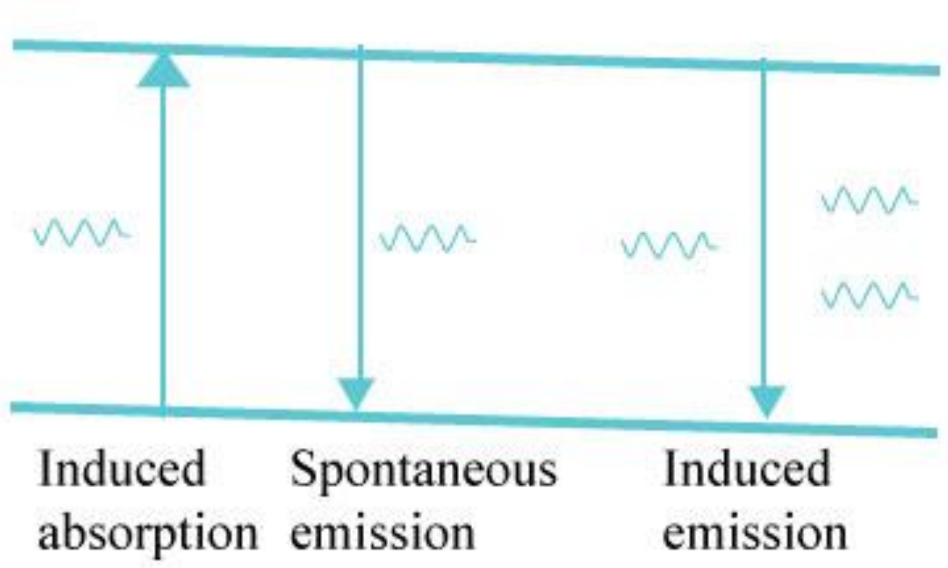


Figure 4.19 How emission and absorption spectral lines originate.

- **collision**

∵ energy transfer is a max when the colliding particles have the same mass. → e^- is more effective than ions to provide energy to atomic electrons.

Ex: Neon signs &mercury-vapor lamps.

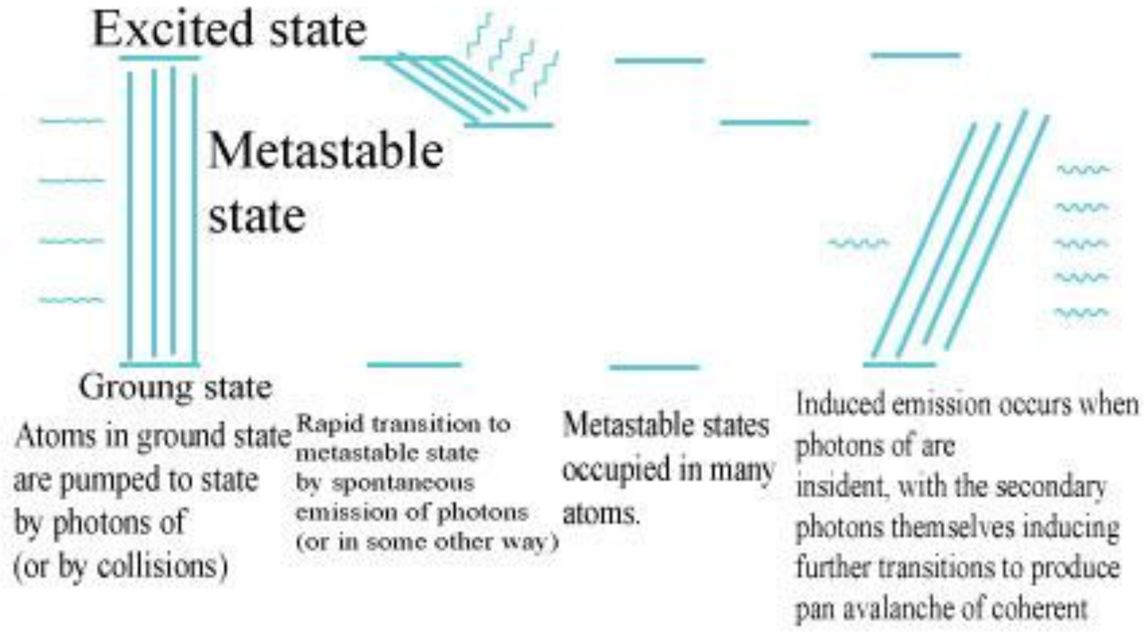


Figure 4.20 The dark lines in an absorption spectrum are never totally dark.

Photon absorption

$H^{n=1} \xrightarrow{\text{absorption}} H^{n=2} \xrightarrow{\text{reradiation}} H^{n=1}$

$h\nu \nearrow$ $\searrow$ $h\nu$ radiation

The resulting excited H atoms reradiate their excitation energy almost at once, but these photons come off in random directions, with only a few in the same direction as the original beam of white light The dark line in an absorption spectrum are never completely black.

V_0: prevent e’ having energies less than a certain minimum from contributing to current I

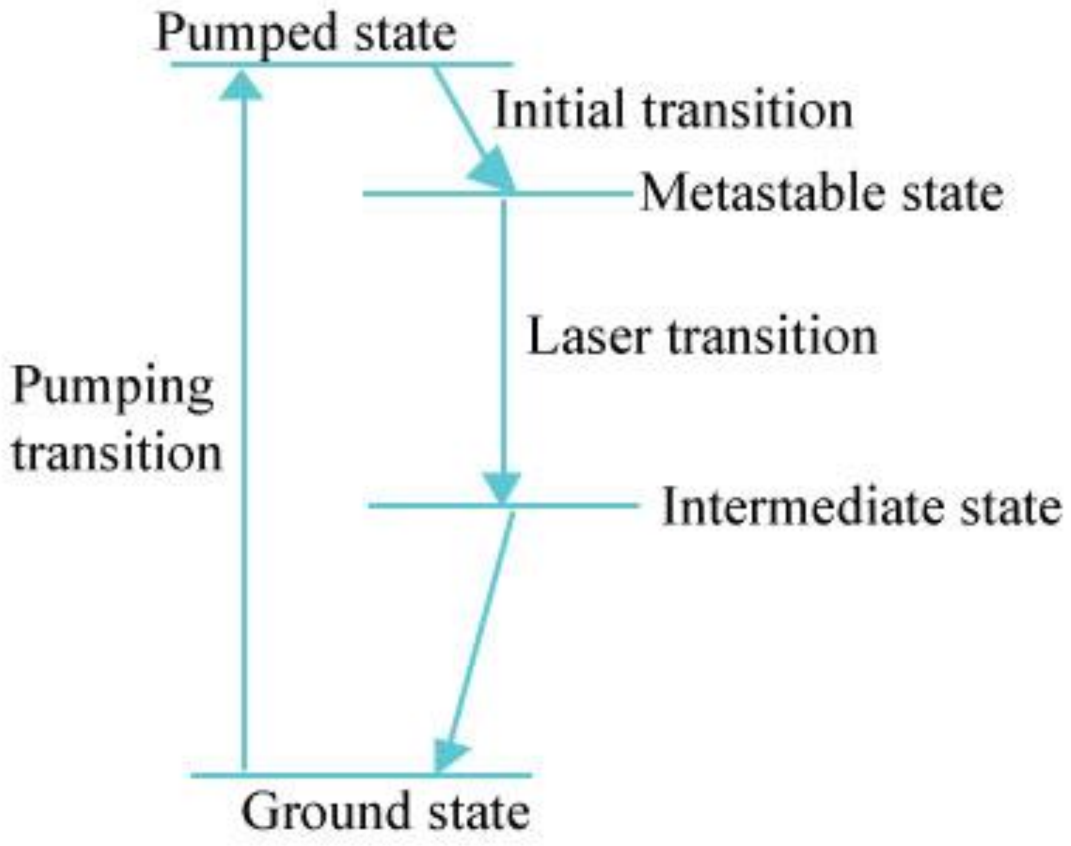


Figure 4.21 Apparatus for the Franck-Hertz experiment.

Franck-Hertz experiment when V↑ → # of e'↑ → I↑ → it is not the case, at some specific V

- If KE is conserved → when e' collides with atoms in vapor, the e' merely bounces off in a new direction.(∵ e' is light → almost no KE loss)
- at a critical V → I↓ → e' colliding with one of atoms gives up some or all of its KE to excite atom to higher energy level.

→ In elastic

- when V↑ again → I↑ again(∵ e' now have enough energy left to reach the plate after an inelastic collision)
- V↑↑ → another sharp drop

  → Excitation of the same energy level in other atoms by e'

* to check critical V is due to atomic energy level → Check emission spectra for Mercury vapor: 4.9eV was required to excite 253.6nm spectra line.

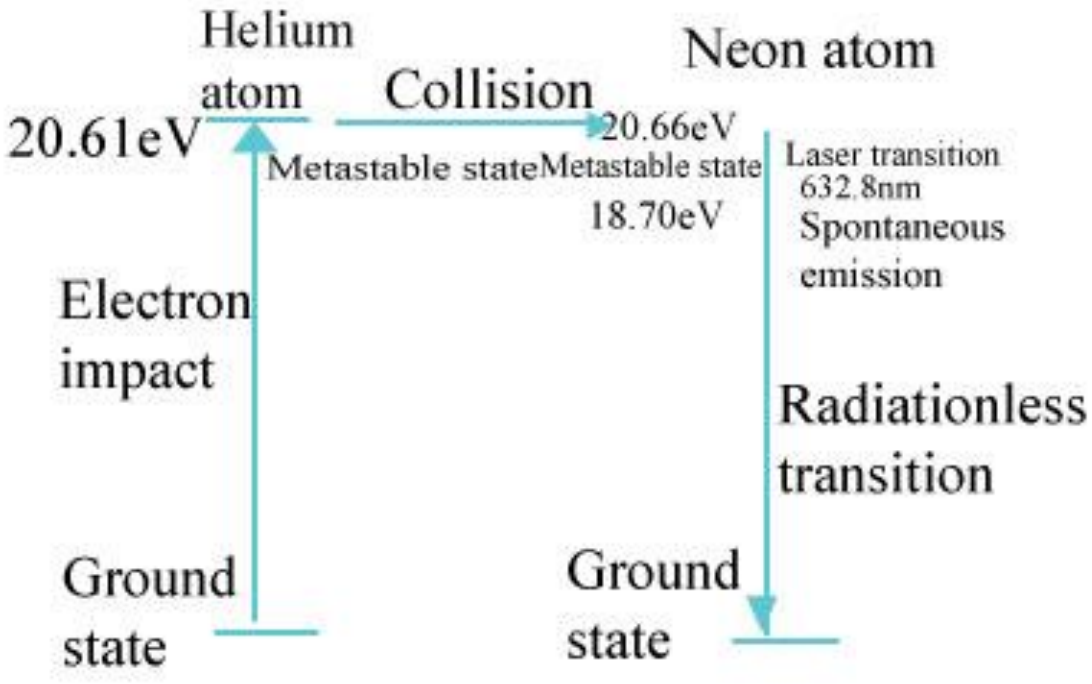


Figure 4.22 An atom exist in a metastable energy level for a longer time before radiating then it can in an ordinary energy level.

Figure 4.23 A laser produces a beam of light whose waves all have the same frequency(monochromatic) and are in phase with one another (coherent). The beam is also well collimated and so spreads out very little, even over long disrances.

【4.9 Laser: light amplification by stimulated emission of radiation 】

.coherent: in phase

.monochromatic: single λ

.very little diverge.

.intense

key for Laser: presence of meta stable states

Figure 4.25 Transitions between two energy levels in an atom can occur by induced absorption, spontaneous emission, and induced emission.

Three kinds of transition between two energy levels.

In induced emission, the radiation waves are exactly in phase with the incident ones → enhanced beam of coherent light.

* $P_{\text{induced emission}}=P_{\text{induced absorption}}$

- three-level laser

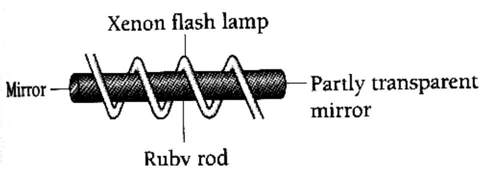
Figure 4.26 The principle of the laser.

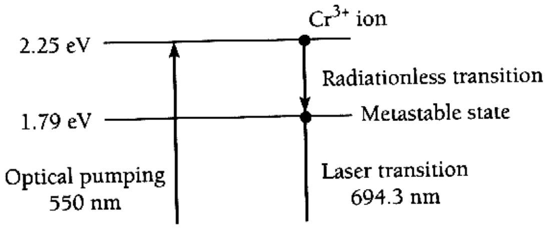
- We want more atoms in Metastable state than in group stat .

→ Shine hν →more induced emission than induced absorption

→ amplification of light.

- population inversion→an assembly of atoms in which the majority are in energy levels above ground state.





- optical pumping: external light photons raise ground-state atoms to excited state → metastable state
- if only two levels→ more photons →upward transition↑

  →induced downward /transition ↑ → half the atoms are in each state

  → $R_{induced}$ emission=$R_{induced}$ absorption

  → No laser amplification

Figure 4.28 A four-level laser.

∵very few atoms in intermediate → modest amount of pumping is enough to populate the metastable state to greater than intermediate state.

- Ruby laser: Al_2O_3, some of Al^{+3} are replaced by Cr^{+3} xenon flash lamp→ excite Cr^{+3} to higher energy

→ Fall to metastable level by losing energy to other ions

→ Photons from spontaneous decay of some Cr^{+3} are reflected back & forth between mirrored ends of ruby rod

→ Stimulating other excited Cr^{+3}

* rod's length=$(\lambda/2)n$ →standing wave.

Figure 4.29 The helium-neon laser. In a four-level laser such as this, continuous operation is possible Helium-neon lasers are commonly used to read bar codes.

He-Ne laser: 10He/1Ne at low pressure.

.use electric discharge →collisions with e' from discharge excite. He&Ne to metastable states

.Some of excited He → transfer energy to ground state Ne in collisions

*The purpose of He is to help achieve a population inversion in →Ne Since the e' impacts the excite He&Ne occur all the time He-Ne laser operates continuously.